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Summary: We study the combinatorial structure of the irreducible characters of the classical groups $GL_n(C)$, $SO_{2n+1}(C)$, $Sp_{2n}(C)$, $SO_{2n}(C)$ and the “non-classical” odd symplectic group $Sp_{2n+1}(C)$, finding new connections to the probabilistic model of Last Passage Percolation (LPP). Perturbing the expressions of these characters as generating functions of Gelfand-Tsetlin patterns, we produce two families of symmetric polynomials that interpolate between characters of $Sp_{2n}(C)$ and $SO_{2n+1}(C)$ and between characters of $SO_{2n}(C)$ and $SO_{2n+1}(C)$. We identify the first family as a one-parameter specialization of Koornwinder polynomials, for which we thus provide a novel combinatorial structure; on the other hand, the second family appears to be new. We next develop a method of Gelfand-Tsetlin pattern decomposition to establish identities between all these polynomials that, in the case of irreducible characters, can be viewed as branching rules. Through these formulas we connect orthogonal and symplectic characters, and more generally the interpolating polynomials, to LPP models with various symmetries, thus going beyond the link with classical Schur polynomials originally found by J. Baik and E. M. Rains [Duke Math. J. 109, No. 2, 205–281 (2001; Zbl 1007.60003)]. Taking the scaling limit of the LPP models, we finally provide an explanation of why the Tracy-Widom GOE and GSE distributions from random matrix theory admit formulations in terms of both Fredholm determinants and Fredholm Pfaffians.

MSC:

05E05 Symmetric functions and generalizations
60C05 Combinatorial probability
60F05 Central limit and other weak theorems
05E10 Combinatorial aspects of representation theory
82B23 Exactly solvable models; Bethe ansatz

Keywords:
symplectic characters; orthogonal characters; interpolating Schur polynomials; last passage percolation; RSK correspondence; Tracy-Widom distributions

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