Emery, M.

Some cases of chaotic representation. (Quelques cas de représentation chaotique.) (French) Zbl 0754.60043

Séminaire de probabilités, Lect. Notes Math. 1485, 10-23 (1991).

[For the entire collection see Zbl 0733.00018.]

The author considers a martingale X with respect to a filtration $(\mathcal{F}_t)$ such that $\langle X,X\rangle_t=t$; $(\mathcal{N}_t^X)$ is the corresponding natural filtration of X. Let S be the disjoint union of all S_n, $n\ge 0$, where $S_n\subset(0,\infty)^n$ is the set of all n-element subsets of $(0,\infty)$, with their elements written in an ascending order, and let λ be the direct sum of the corresponding Lebesgue measures. There is a linear isometric mapping $f\to\int f dX$ from $L^2(S,\lambda)$ to L^2; let $H(X)$ be its image. A more general stochastic integral $\int\chi_{\mathcal{A}_T}fdX$ is defined, for every $f\in L^2(\mathcal{B}(S)\otimes\mathcal{F}_T)$ null outside $\mathcal{A}_T=\{(A,\omega);A\subset(T(\omega),\infty)\}$, where T is a stopping time; in its definition (X_{T+t}) is used. Let $H^T(X)$ be its image. If $\mathcal{N}_0^X$ is trivial and for every $(\mathcal{N}_t^X)$-martingale M_t there is an $\mathcal{N}^X$-previsible ϕ with $dM_t=\phi_t dX_t$, then X is said to have PRP (propriété de représentation prévisible), while, if $H(X)$ is the whole $L^2(\mathcal{N}_\infty^X)$, X is said to have PRC (... chaotique).

The main purpose of this paper is to give new examples of X's having PRC (there are 7 quotations with such examples). The first is $Z_t=X_t$ for $t\le T$, $Z_t=X_T+Y_{t-T}-Y_0$ for $t\ge T$, where X, Y are independent, both having PRC, and T is an $(\mathcal{N}_t^X)$-stopping time. The second is a Y for which there exist X^n having PRC and $(\mathcal{N}_t^{X^n})$-stopping times T_n such that $Y=X^n$ on $[0,T_n]$ and $\sup T_n=\infty$. The third is an X having PRP, with $\langle X,X\rangle_t=t$, $d[X,X]_t=dt+\phi_t dX_t$, ϕ being previsible, nowhere null, with $\int\chi_{[0,t]}(s)\phi_s^{-2}ds<\infty$ for all t. The fourth is X from the solution (X,E) (its existence and unicity in law are shown) of $d[X,X]_t=dt+dE_t$, $dE_t=E_{t-}\lambda dX_t$, $X_0=x$, $E_0=e$, where λ, x, e are constants. The proofs begin by "Proposition 1", relative to two martingales X, Y with $\langle X,X\rangle_t=\langle Y,Y\rangle_t=t$ and $X=Y$ on $[0,T]$, T being a stopping time. In the last statement of this proposition, X is PRC and T is an $(\mathcal{N}_t^X)$-stopping time. Proposition 1 involves also $g=C_T(U,X)$, $h=C(U,X)$ for $U\in L^2$, where the projections of U on $H^T(X)$ and $H(X)$ are $\int\chi_{\mathcal{A}_T}gdX$, $\int hdX$, respectively. The paper finishes with other two results. The first expresses $C(U,X)$ using $C(V,X)$'s with $\mathcal{F}_T$-measurable V's and $C_T(U,X)$ and the second proves that a sufficient condition for $U\in H(X)$ is that X has PRP and $\int E(C_{\inf A-}(A)^2)\lambda(dA)<\infty$, where $dC_t(A)=\Gamma_t(A)dX_t$ and, for $A=\{\cdots<b<c\}$, $C_t(A)$ is $E(U;\mathcal{F}_t)$ for $t\ge c$, and $E(\Gamma_c(A);\mathcal{F}_t)$ for $t\in[b,c)$ etc.

Reviewer: I.Cuculescu (Bucureşti)

MSC:

60G44 Martingales with continuous parameter
60H05 Stochastic integrals

Keywords:

property of chaotic representation; property of previsible representation; recollement; martingale; stochastic integral; existence and unicity in law

Full Text: Numdam Numdam EuDML