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A distributional approach to fractional Sobolev spaces and fractional variation: asymptotics II. (English) [zbMATH identifier Zbl 07547261]

Summary: We continue the study of the space BVα(Rn) of functions with bounded fractional variation in Rn and of the distributional fractional Sobolev space Sα,p(Rn), with p ∈ [1, +∞] and α ∈ (0, 1), considered in the previous works [28, 27]. We first define the space BV0(Rn) and establish the identifications BV0(Rn) = H1(Rn) and Sα,p(Rn) = Lα,p(Rn), where H1(Rn) and Lα,p(Rn) are the (real) Hardy space and the Bessel potential space, respectively. We then prove that the fractional gradient ∇α strongly converges to the Riesz transform as α → 0+ for H1 ∩ Wα,1 and Sα,p functions. We also study the convergence of the L1-norm of the α-rescaled fractional gradient of Wα,1 functions. To achieve the strong limiting behavior of ∇α as α → 0+, we prove some new fractional interpolation inequalities which are stable with respect to the interpolating parameter.

MSC:
26A33 Fractional derivatives and integrals
26B30 Absolutely continuous real functions of several variables, functions of bounded variation
28A33 Spaces of measures, convergence of measures
47G40 Potential operators

Full Text: DOI

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