Let $I$ and $J$ be two intervals of positive integers of the same length. A bijection $f : I \to J$ is called a **coprime matching** if $f(i)$ is relatively prime to $i$ for every $i \in I$. In the article under review, the author proves that there is a positive constant $c$ such that if $n$ is sufficiently large, $m > c(\log n)^2$, and $I, J \subset \{1, 2, \ldots, n\}$ with $|I| = |J| = 2m$, then there is a coprime matching of $I$ and $J$. This result, Theorem 1 of the paper, improves on a theorem of Bohman and Peng, which requires $m > \exp(C(\log \log n)^2)$ where $C$ is a positive constant. The proof proceeds by reducing the problem to one concerning the number of $2$-coprime pairs (that is, the only common prime factor of the pair is $2$) between elements of $I$ and $J$. The argument then relies on a result of Iwaniec asserting that, typically, many elements of a subset $S \subset I$ are $2$-coprime to many elements of $J$.

Interestingly, Theorem 1 (as well as the Bohman-Peng result) has applications to the lonely runner conjecture, which states that if $v_1, \ldots, v_n$ are distinct positive integers, then there exists $t \in \mathbb{R}$ such that no quantity of the form $v_i t$ is strictly within $1/(n+1)$ of any integer. Theorem 1 establishes this conjecture in the case in which each $v_i$ is at most $2n - c'(\log n)^2$.

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MSC:

11B75 Other combinatorial number theory
11A25 Arithmetic functions; related numbers; inversion formulas

Keywords:

lonely runner conjecture; coprime mappings

Full Text: arXiv Link

References:


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