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On convex and concave sequences and their applications. (English) Zbl 07566815

Summary: The aim of this paper is to introduce and to investigate the basic properties of $q$ convex, $q$-affine and $q$-concave sequences and to establish their surprising connection to Chebyshev polynomials of the first and of the second kind. One of the main results shows that $q$ concave sequences are the pointwise minima of $q$-affine sequences. As an application, we consider a nonlinear selfmap of then-dimensional space and prove that it has a unique fixed point. For the proof of this result, we introduce a new norm on the space in terms of a $q$-concave sequence and show that the nonlinear operator becomes a contraction with respect to this norm, and hence, the Banach Fixed Point theorem can be applied.

MSC:
26A51 Convexity of real functions in one variable, generalizations
39B62 Functional inequalities, including subadditivity, convexity, etc.

Keywords:
$q$-convex sequence; $q$-concave sequence; $q$-affine sequence; Chebyshev polynomials of first and second kind; contraction

Full Text: DOI

References:


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