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The arithmetic basilica: a quadratic PCF arboreal Galois group. (English) [Zbl 07569757] J. Number Theory 238, 842-868 (2022)

Summary: The arboreal Galois group of a polynomial $f$ over a field $K$ encodes the action of Galois on the iterated preimages of a root point $x_0 \in K$, analogous to the action of Galois on the $p$-power torsion of an abelian variety. We compute the arboreal Galois group of the postcritically finite polynomial $f(z) = z^2 - 1$ when the field $K$ and root point $x_0$ satisfy a simple condition. We call the resulting group the arithmetic basilica group because of its relation to the basilica group associated with the complex dynamics of $f$. For $K = \mathbb{Q}$, our condition holds for infinitely many choices of $x_0$.

MSC:
37P05 Arithmetic and non-Archimedean dynamical systems involving polynomial and rational maps
11R32 Galois theory
14G25 Global ground fields in algebraic geometry

Keywords:
arithmetic dynamics; arboreal Galois representations

Full Text: DOI

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