Andersen, Kenneth F.

Weighted generalized Hardy inequalities for nonincreasing functions. (English)

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The strong or weak weighted (p,q) inequalities of Hardy's operators $P(f \to \frac{1}{x}\int_0^x f(t)dt)$ and $P'(f \to \int_x^\infty \frac{f(t)}{t}dt)$ restricted to nonnegative nonincreasing functions occur naturally in certain rearrangement inequalities. The paper is concerned with the following generalization of P and P', i.e., $T : f \to Tf(x) = \int_0^\infty a(t)f(xt)dt$, with $a(t)$ a nonnegative mesurable function. The main result (Theorem 1) is the characterization of the weak $(L^p(v), L^q(u))$ boundedness of T for $0 < p,q < \infty$. The preceding weak type characterization can be used to obtain some strong type characterizations of T. For example (a part of Theorem 2), in the case $p = q$ and when $A(ts) \le BA(t)A(s)$, $0 < t,s \le 1$, then T's $(L^p(v), L^q(u))$ boundedness can be characterized by the simple condition

$$\int_r^\infty A\left(\frac{r}{x}\right)^p u(x)dx \le KU(r), \quad \forall r > 0,$$

where $A(x) = \int_0^x a(t)dt$, $U(x) = \int_0^x u(t)dt$.

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MSC:

26D15	Inequalities for sums, series and integrals	Cited in **1** Review
42B25	Maximal functions, Littlewood-Paley theory	Cited in **12** Documents

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weighted inequalities; weak type inequalities; Hardy's operators; rearrangement inequalities

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