Arutyunov, A. V.
On the theory of quadratic mappings in Banach spaces. (English. Russian original)

Let $X$ be a Banach space and $A : X \times X \to E^n$ be a bilinear continuous mapping with values in $n$-dimensional space and such that $A(x_1, x_2) = A(x_2, x_1) \forall x_i \in X$. The mapping $Q : X \to E^n$ defined by $Q(x) = A(x, x)$ is called a quadratic mapping. Let $C$ be a closed convex cone in $E$ and $H = \{x \in X : Q(x) \in C\}$. A quadratic mapping $Q$ is said to be strongly 2-regular (with respect to the cone $C$) if there exists an $\varepsilon > 0$ such that $A(x, D_X) \supseteq \{z \in E^n : z \in \text{Lin}C^0, |z| \leq \varepsilon\}$ for all $x$ in the unit sphere of $X$ and $\rho(Q(x), C) \leq \varepsilon$, where $D_X$ is the unit ball in $X$, $C^0 = \{z \in E^n : \langle z, e \rangle \leq 0 \forall e \in C\}$ is the cone dual to $C$, Lin is the linear span of a set, and $\rho$ is the distance from a point to a set. The author describes the weak closure of $H$ and gets a criterion for a quadratic mapping to be strongly 2-regular.

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weak closure; Fréchet-differentiable; bilinear continuous mapping with values in $n$-dimensional space; criterion for a quadratic mapping to be strongly 2-regular