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Nonlocal games with noisy maximally entangled states are decidable. (English) Zbl 07572359
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Summary: This paper considers a special class of nonlocal games \((G, \psi)\), where \(G\) is a two-player one-round game, and \(\psi\) is a bipartite state independent of \(G\). In the game \((G, \psi)\), the players are allowed to share arbitrarily many copies of \(\psi\). The value of the game \((G, \psi)\), denoted by \(\omega^*(G, \psi)\), is the supremum of the winning probability that the players can achieve with arbitrarily many copies of preshared states \(\psi\). For a noisy maximally entangled state \(\psi\), a two-player one-round game \(G\) and an arbitrarily small precision \(\epsilon > 0\), this paper proves an upper bound on the number of copies of \(\psi\) for the players to win the game with a probability \(\epsilon\) close to \(\omega^*(G, \psi)\). A noisy maximally entangled state is a two-qudit state with both marginals being completely mixed states and the maximal correlation being less than 1. In particular, it includes \((1 - \epsilon)|\Psi_m\rangle\langle\Psi_m| + \epsilon \frac{1}{\sqrt{m}} \otimes \frac{1}{\sqrt{m}}\) for \(\epsilon > 0\), where \(|\Psi_m\rangle = \frac{1}{\sqrt{m}} \sum_{i=0}^{m-1} |m, m\rangle\) is an \(m\)-dimensional maximally entangled state. Hence, it is feasible to approximately compute \(\omega^*(G, \psi)\) to an arbitrary precision. Recently, a breakthrough result by Ji et al. showed that it is undecidable to approximate the values of nonlocal games to a constant precision, when the players preshare arbitrarily many copies of perfect maximally entangled states, which implies that \(\text{MIP}^* = \text{RE}\). In contrast, our result implies the hardness of approximating nonlocal games collapses when the preshared maximally entangled states are noisy. The paper develops a theory of Fourier analysis on matrix spaces by extending a number of techniques in Boolean analysis and Hermitian analysis to matrix spaces. We establish a series of new techniques, such as a quantum invariance principle and a hypercontractive inequality for random operators, which we believe have further applications. (A corrected version is attached.)

MSC:
81P40 Quantum coherence, entanglement, quantum correlations
68Q12 Quantum algorithms and complexity in the theory of computing
81P68 Quantum computation
91A44 Games involving topology, set theory, or logic

Keywords:
nonlocal games; Fourier analysis; hypercontractive inequality; quantum invariance principle

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References:
[21] A. Connes, Classification of injective factors cases \(\text{II}_1, \text{II}_\infty, \text{III}_\lambda, \lambda\neq 1\), Ann. of Math. (2), 104 (1976), pp. 73-115, http://www.jstor.org/stable/1971057. · Zbl 0317.94025
[29] P. Gacs and J. Körner, Common information is far less than mutual information, Problems of Control and Information Theory, 2 (1973), pp. 149-162. · Zbl 0317.94025
  · Zbl 1127.68405


