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The braid group $B_{n,m}(\mathbb{R}P^2)$ and the splitting problem of the generalised Fadell-Neuwirth short exact sequence. (English) Zbl 07573824

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Summary: Let $n$, $m \in \mathbb{N}$, and let $B_{n,m}(\mathbb{R}P^2)$ be the set of $(n+m)$-braids of the projective plane whose associated permutation lies in the subgroup $S_n \times S_m$ of the symmetric group $S_{n+m}$. We study the splitting problem of the following generalisation of the Fadell-Neuwirth short exact sequence:

$$1 \to B_n(\mathbb{R}P^2 \setminus \{x_1, \ldots, x_n\}) \to B_{n,m}(\mathbb{R}P^2) \to B_n(\mathbb{R}P^2) \to 1,$$

where the map $\tilde{q}$ can be considered geometrically as the epimorphism that forgets the last $m$ strands, as well as the existence of a section of the corresponding fibration $q : F_{n+m}(\mathbb{R}P^2)/S_n \times S_m \to F_n(\mathbb{R}P^2)/S_n$, where we denote by $F_n(\mathbb{R}P^2)$ the $n$th ordered configuration space of the projective plane $\mathbb{R}P^2$.

Our main results are the following: if $n = 1$ the homomorphism $\tilde{q}$ and the corresponding fibration $q$ admits no section, while if $n = 2$, then $\tilde{q}$ and $q$ admit a section. For $n \geq 3$, we show that if $\tilde{q}$ and $q$ admit a section then $m \equiv 0$, $(n-1)^2 \mod n(n-1)$. Moreover, using geometric constructions, we show that the homomorphism $\tilde{q}$ and the fibration $q$ admit a section for $m = kn(2n-1)(2n-2)$, where $k \geq 1$, and for $m = 2n(n-1)$. In addition, we show that for $m \geq 3$, $B_{n,m}(\mathbb{R}P^2 \setminus \{x_1, \ldots, x_n\})$ is not residually nilpotent and for $m \geq 5$, it is not residually solvable.

MSC:
- 54-XX General topology
- 55-XX Algebraic topology
- 57-XX Manifolds and cell complexes

Keywords:
- surface braid group; group presentation; Fadell-Neuwirth short exact sequence; section problem; fibration; residually nilpotent; residually solvable

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