Coleman, Robert F.; **Gross, Benedict H.**
p-adic heights on curves. (English) Zbl 0758.14009
Algebraic number theory - in honor of K. Iwasawa, Proc. Workshop Iwasawa Theory Spec. Values L-Funct., Berkeley/CA (USA) 1987, Adv. Stud. Pure Math. 17, 73-81 (1989).

[For the entire collection see Zbl 0721.00006.]

k denotes a non-archimedean local field of characteristic zero and $\chi : k^* \to \mathbb{Q}_p$ denotes a continuous character. Let J be the Jacobian variety of a curve X over k (having a k-rational point). The aim of the paper is to construct a p-adic height pairing on J.

In the case that the residue characteristic of k is different from p, arithmetic intersection theory is used to produce a unique pairing $\langle a, b\rangle$, with values in $\mathbb{Q}_p$, defined on relatively prime divisors a and b on X (defined over k) and satisfying: continuous, symmetric, bi-additive and $\langle (f), b\rangle = \chi(f(b))$ for $f \in k(X)^*$. In the case $k \supset \mathbb{Q}_p$, a rigid analytic analysis of differentials of the third kind and the de Rham cohomology is made to arrive at a definition of the pairing. The pairing which is constructed depends on a suitable choice of a direct sum decomposition $H^1_{DR}(X/k) = H^0(X, \Omega_X) \oplus W$. In case X has a good ordinary reduction one can take the unit root space as a choice for W. With this choice the pairing coincides with the canonical p-adic height pairings constructed by *P. Schneider* [Invent. Math. 69, 401–409 (1982; Zbl 0509.14048 and 79, 329–374 (1985; Zbl 0571.14021)] and by *B. Mazur* and *J. Tate* in Arithmetic and geometry, Pap. dedic. Shafarevich, Vol. I. Arithmetic, Prog. Math. 35, 195–237 (1983; Zbl 0574.14036)]. A proof of the last statement is given in the sequel of this paper [*R. F. Coleman*, “The universal vectorial bi-extension and p-adic heights”, Invent. Math. 103, No. 3, 631–650 (1991; Zbl 0763.14009)].

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MSC:

14G20	Local ground fields in algebraic geometry
14G40	Arithmetic varieties and schemes; Arakelov theory; heights
14H25	Arithmetic ground fields for curves

Keywords:

p-adic height pairing; Jacobian variety; arithmetic intersection theory; differentials of the third kind; de Rham cohomology