Summary: We construct a family of infinite simple groups that we call *twisted Brin-Thompson groups*, generalizing Brin’s higher-dimensional Thompson groups $sV$ for $s \in \mathbb{N}$. We use twisted Brin-Thompson groups to prove a variety of results regarding simple groups. For example, we prove that every finitely generated group embeds quasi-isometrically as a subgroup of a two-generated simple group, strengthening a result of Bridson. We also produce examples of simple groups that contain every $sV$ and hence every right-angled Artin group, including examples of type $F_\infty$ and a family of examples of type $F_{n-1}$ but not of type $F_n$ for arbitrary $n \in \mathbb{N}$. This provides the second known infinite family of simple groups distinguished by their finiteness properties.

**MSC:**

- 20F65 Geometric group theory
- 20E32 Simple groups
- 57M07 Topological methods in group theory

**Keywords:**

Thompson group; finiteness properties; simple group; right-angled Artin group; quasi-isometry; oligomorphic; Cantor space

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