Summary: The structure group of an involutive set-theoretic solution to the Yang-Baxter equation is a generalized radical ring called a brace. The concept of brace is extended to that of a quasiring where the adjoint group is just a monoid. It is proved that a special class of lattice-ordered quasirings characterizes the divisor group $A$ of a smooth non-commutative curve $X$. The multiplicative monoid $A^\times$ of $A$ is related to the additive group by a bijective 1-cocycle. Extending previous results on non-commutative arithmetic, the elements of $A$ are represented as a class $\Phi(X)$ of self-maps of a universal cover of $X$. For affine subsets $U$ of $X$, the regular functions on $U$ form a hereditary order such that the monoid of fractional ideals embeds into $A^\times$ as the class of monotone functions in $\Phi(U)$. The unit group of $A$ is identified with the annular symmetric group, which occurred in connection with quasi-Garside groups of Euclidean type. The main part of the paper is self-contained and provides a quick approach to non-commutative prime factorization and its relationship to braces.

MSC:

16T25 Yang-Baxter equations
14A22 Noncommutative algebraic geometry
20F36 Braid groups; Artin groups
17B37 Quantum groups (quantized enveloping algebras) and related deformations
16W30 Hopf algebras (associative rings and algebras) (MSC2000)
06F15 Ordered groups
20M30 Representation of semigroups; actions of semigroups on sets
11M55 Relations with noncommutative geometry

Keywords:
bijective cocycle; brace; hereditary order; non-commutative curve; prime factorization; divisor group; right $\ell$-group; quasi-Garside group

Full Text: DOI

References:

V. G. Drinfeld, On some unsolved problems in quantum group theory, in: Quantum Groups (Leningrad, 1990), Lecture Notes
Zbl 0238.20034
E. Jespers and J. Okniński, Monoids and groups of I-type, Algebras Represent. Theory 8 (2005), 709-729. - Zbl 1423.20032
Zbl 1091.20024
Zbl 0972.20001
Zbl 1216.16023
Zbl 0211.05701
S. C. Featherstonhaugh, A. Caranti and L. N. Childs, Abelian Hopf Galois structures on prime-power Galois field extensions,
F. Chouraqui, Garside groups and Yang-Baxter equation, Comm. Algebra 38 (2010), 4441-4460. - Zbl 1216.16023
E. Jespers and J. Okniński, Monoids and groups of I-type, Algebras Represent. Theory 8 (2005), 709-729. - Zbl 1091.20024
J. McCammond, Dual euclidean Artin groups and the failure of the lattice property, J. Algebra 437 (2015), 308-343. - Zbl 1343.20039