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Orthogonal polynomials defined by self-similar measures with overlaps. (English)
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Summary: We study orthogonal polynomials with respect to self-similar measures, focusing on the class of infinite Bernoulli convolutions, which are defined by iterated function systems with overlaps, especially those defined by the Pisot, Garsia, and Salem numbers. By using an algorithm of Mantica, we obtain graphs of the coefficients of the 3-term recursion relation defining the orthogonal polynomials. We use these graphs to predict whether the singular infinite Bernoulli convolutions belong to the Nevai class. Based on our numerical results, we conjecture that all infinite Bernoulli convolutions with contraction ratios greater than or equal to 1/2 belong to Nevai’s class, regardless of the probability weights assigned to the self-similar measures.

MSC:
42C05 Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis
28A25 Integration with respect to measures and other set functions
28A80 Fractals
33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

Keywords:
orthogonal polynomial; self-similar measure with overlaps; Nevai class

Full Text: DOI

References: