Let $S$ be an $n \times n$ Hermitian matrix and let $\sigma_\sigma(S)$ denote the ordered vector of $(\lambda_1, \ldots, \lambda_n)$ of its eigenvalues with $\lambda_1 \leq \cdots \leq \lambda_n$. A well known theorem states (*): if $\sigma_\sigma(S) = (\lambda_1, \ldots, \lambda_n)$ then for each vector $v \in \mathbb{C}^n$ we have $\sigma_\sigma(S + vv^*) = (\mu_1, \ldots, \mu_n)$ where $\lambda_i \leq \mu_i$ for each $i$ and $\mu_i \leq \lambda_{i+1}$ for $i \neq n$; conversely, for any choice of $(\mu_1, \ldots, \mu_n) \in \mathbb{R}$ satisfying these conditions there is an appropriate $v$ (see, for example, [R. A. Horn and C. R. Johnson, Matrix analysis. 2nd ed. Cambridge: Cambridge University Press (2013; Zbl 1267.15001)]). The object of this paper is to explore such relationships of spectral interlacing further.

Fix a unitary matrix $Q$ such that $Q^* SQ = \text{diag}(\lambda_1, \ldots, \lambda_n)$. Then the $i$-th column $Q$ is an eigenvector for the eigenvalue $\lambda_i$ and we define $O_Q$ to be the set of all $Qp$ where $p \in \mathbb{R}^n$ has nonnegative entries. Define $F : O_Q \to P_F$ by $v \mapsto \sigma_\sigma(S + vv^*)$ where $P_F := [\lambda_1, \lambda_2] \times [\lambda_2, \lambda_3] \times \cdots \times [\lambda_n, \infty]$. Finally for $r > 0$ let $S(r) := \{v \in \mathbb{C}^n \mid \|v\| = r\}$ and $P_F(r) := \{\mu \in P_F \mid \sum_j \mu_j = r^2 + \sum_j \lambda_j\}$.

Then the restriction of $F$ to vectors of length $r$ defines a function $F^r : O_Q \cap S(r) \to P_F(r)$. The authors prove that $F$ and the functions $F^r$ are homeomorphisms and are diffeomorphisms between the interiors of the domain and image. Clearly (*) is a consequence of this result. A similar result is proved for Cauchy’s theorem on the interlacing of the eigenvalues of $S$ with those of the $(n+1) \times (n+1)$ matrices of the form

$$T(v,e) := \begin{bmatrix} S & v \\ v^* & e \end{bmatrix}.$$ 

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MSC:

15B57 Hermitian, skew-Hermitian, and related matrices
15A18 Eigenvalues, singular values, and eigenvectors
15A04 Linear transformations, semilinear transformations

Keywords:
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