Kang, Hongchao; Zhang, Meijuan

Summary: This paper proposes and analyzes two affordable and efficient quadrature rules for the numerical approximation of the oscillatory Bessel transform \( \int_0^b x^\alpha (b-x)^\beta f(x) J_\nu(\omega x^\gamma) dx \) with algebraic singularities, where \( b, \alpha, \beta, \nu, \omega, \gamma \) denote the given constants. Firstly, we derive the explicit formula and asymptotic estimation of the generalized moments \( \int_0^b x^{\alpha'} (b-x)^{\beta'} J_\nu(\omega x^\gamma) dx \) with \( \alpha', \beta' > -1 \) by means of the Meijer G function. Furthermore, we design a modified Filon-type method based on a two-endpoint Taylor interpolation polynomial. In particular, we also give a more efficient Clenshaw-Curtis-Filon-type method in view of a special Hermite interpolation polynomial at the Clenshaw-Curtis points. Moreover, this method is easily implemented by the fast Fourier transform and fast computation of the modified moments. The useful homogeneous recurrence relation of the required modified moments is derived by the Bessel equation and the properties of the Chebyshev polynomial. Importantly, the rigorous error analyses in inverse powers of \( \omega \) for the proposed numerical methods are carried out in details. Some primary numerical experiments can confirm our theoretical analysis, and verify the accuracy and efficiency of the proposed numerical methods.

MSC:
65D32 Numerical quadrature and cubature formulas
41A55 Approximate quadratures

Keywords:
singular Bessel transform; two-point Taylor interpolation polynomial; modified Filon-type method; special Hermite interpolation polynomial; Clenshaw-Curtis-Filon-type method; error analysis

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