Summary: The determinantal complexity of a polynomial \( P \in \mathbb{F}[x_1, \ldots, x_n] \) over a field \( \mathbb{F} \) is the dimension of the smallest matrix \( M \) whose entries are affine functions in \( \mathbb{F}[x_1, \ldots, x_n] \) such that \( P = \text{Det}(M) \). We prove that the determinantal complexity of the polynomial \( \sum_{i=1}^{n} x_i^n \) is at least \( 1.5n - 3 \). For every \( n \)-variate polynomial of degree \( d \), the determinantal complexity is trivially at least \( d \), and it is a long-standing open problem to prove a lower bound which is super linear in \( \max\{n, d\} \). Our result is the first lower bound for any explicit polynomial which is bigger by a constant factor than \( \max\{n, d\} \), and improves upon the prior best bound of \( n + 1 \), proved by Alper et al. (2017) for the same polynomial.

MSC:

68Q06 Networks and circuits as models of computation; circuit complexity
68Q15 Complexity classes (hierarchies, relations among complexity classes, etc.)
68Q17 Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.)

Keywords:
determinantal complexity; algebraic complexity theory; lower bounds; algebraic circuits

Software:

GitHub

Full Text: DOI arXiv

References:


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