Sturm, Karl-Theodor

Random Riemannian geometry in 4 dimensions. (English) Zbl 07605724


Summary: We construct and analyze conformally invariant random fields on 4-dimensional Riemannian manifolds \( (M, g) \). These centered Gaussian fields \( h \), called \textit{co-biharmonic Gaussian fields}, are characterized by their covariance kernels \( k \) defined as the integral kernel for the inverse of the Paneitz operator

\[
p = \frac{1}{8\pi^2} \left[ \Delta^2 + \text{div} \left( 2\text{Ric} - \frac{2}{3}\text{scal} \right) \nabla \right].
\]

The kernel \( k \) is invariant (modulo additive corrections) under conformal transformations, and it exhibits a precise logarithmic divergence

\[ |k(x, y) - \log \frac{1}{d(x, y)}| \leq C. \]

In terms of the co-biharmonic Gaussian field \( h \), we define the \textit{quantum Liouville measure}, a random measure on \( M \), heuristically given as

\[ d\mu(x) := e^{\gamma h(x) - \frac{\gamma^2}{2} k(x, x)} d\text{vol}_{g}(x), \]

and rigorously obtained a.s. for \( |\gamma| < \sqrt{8} \) as weak limit of the RHS with \( h \) replaced by suitable regular approximations \( (h_{\ell})_{\ell \in \mathbb{N}} \). For the flat torus \( M = \mathbb{T}^4 \), we provide discrete approximations of the Gaussian field and of the Liouville measures in terms of semi-discrete random objects, based on Gaussian random variables on the discrete torus and piecewise constant functions in the isotropic Haar system.

For the entire collection see [Zbl 1493.11005].

MSC:

- 60G15 Gaussian processes
- 58J65 Diffusion processes and stochastic analysis on manifolds
- 31C25 Dirichlet forms

Keywords:

random Riemannian geometry; Gaussian field; conformally invariant; Paneitz operator; bi-Laplacian; biharmonic; membrane model; quantum Liouville measure

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