

de la Peña, Victor H.

Decoupling and Khintchine's inequalities for U -statistics. (English) Zbl 0761.60014
Ann. Probab. 20, No. 4, 1877-1892 (1992).

Summary: We introduce a fairly general decoupling inequality for U -statistics. Let $\{X_i\}$ be a sequence of independent random variables in a measurable space (S, \mathcal{S}) , and let $\{\tilde{X}_i\}$ be an independent copy of $\{X_i\}$. Let $\Phi(x)$ be any convex increasing function for $x \geq 0$. Let Π_{ij} be families of functions of two variables taking $(S \times S)$ into a Banach space $(D, \|\cdot\|)$. If the $f_{ij} \in \Pi_{ij}$ are Bochner integrable and

$$\max_{1 \leq i \neq j \leq n} E\Phi \left(\sup_{f_{ij} \in \Pi_{ij}} \|f_{ij}(X_i, X_j)\| \right) < \infty,$$

then, under measurability conditions,

$$E\Phi \left(\sup_{\mathbf{f} \in \Pi} \left\| \sum_{1 \leq i \neq j \leq n} f_{ij}(X_i, X_j) \right\| \right) \leq E\Phi \left(8 \sup_{\mathbf{f} \in \Pi} \left\| \sum_{1 \leq i \neq j \leq n} f_{ij}(X_i, \tilde{X}_j) \right\| \right),$$

where $\mathbf{f} = (f_{ij}, 1 \leq i \neq j \leq n)$ and $\Pi = (\Pi_{ij}, 1 \leq i \neq j \leq n)$. In the case where Π is a family of functions of two variables satisfying $f_{ij} = f_{ji}$ and $f_{ij}(X_i, X_j) = f_{ij}(X_j, X_i)$, the reverse inequality holds (with a different constant). As a corollary, we extend Khintchine's inequality for quadratic forms to the case of degenerate U -statistics. A new maximal inequality for degenerate U -statistics is also obtained. The multivariate extension is provided.

MSC:

60E15 Inequalities; stochastic orderings

Cited in **2** Reviews
Cited in **32** Documents

Keywords:

[Khintchine's inequalities](#); [decoupling inequality](#); [U-statistics](#); [degenerate U-statistics](#)

Full Text: [DOI](#)