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Algebraic foliations and derived geometry: the Riemann-Hilbert correspondence. (English)


Summary: This is the first in a series of papers about foliations in derived geometry. After introducing derived foliations on arbitrary derived stacks, we concentrate on quasi-smooth and rigid derived foliations on smooth complex algebraic varieties and on their associated formal and analytic versions. Their truncations are classical singular foliations defined in terms of differential ideals in the algebra of forms. We prove that a quasi-smooth rigid derived foliation on a smooth complex variety $X$ is formally integrable at any point, and, if we suppose that its singular locus has codimension $\geq 2$, its analyticification is a locally integrable singular foliation on the associated complex manifold $X^h$. We then introduce the derived category of perfect crystals on a quasi-smooth rigid derived foliation on $X$, and prove a Riemann-Hilbert correspondence for them when $X$ is proper. We discuss several examples and applications.

MSC:

37F75 Dynamical aspects of holomorphic foliations and vector fields
14F40 de Rham cohomology and algebraic geometry
14F05 Sheaves, derived categories of sheaves, etc. (MSC2010)
14A20 Generalizations (algebraic spaces, stacks)

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References:


