Krüskemper, Martin

Algebraic number field extensions with prescribed trace form. (English) Zbl 0762.11014

J. Number Theory 40, No. 1, 120-124 (1992).

Let L/F be a finite extension of algebraic number fields. A quadratic form over F is called positive, if the signature of this form is nonnegative for all orderings of F. The trace form $Tr_{L/F}\langle 1\rangle$ is a positive form for example. In the paper under review the following question of Conner and Perlis is investigated: Which positive quadratic forms over F are isometric to a trace form of some field extension L/F? Definition: (tr n) holds, if for every number field F every positive form φ over F with $\dim\varphi = n$ is isometric to a trace form of some field extension L/F with $[L:F] = n$.

The main result of the paper is the following Theorem: Let $n \geq 4$. Then (i) if (tr n) holds then (trnm) holds for all $m \in \mathbb{N}$, (ii) (tr$n$) holds if n is divisible by 2 or 3.

Remark: The question in the case of quadratic forms in dimension 2, 3 and 4 was already answered by Conner and Perlis. For the proof some number theory is needed, for instance the approximation theorem, Dirichlet's density theorem and the ramification of prime ideals in L/F.

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MSC:

11E12 Quadratic forms over global rings and fields
11R21 Other number fields
12F05 Algebraic field extensions

Keywords:

trace form; positive quadratic forms; field extension

Full Text: DOI

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