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Local error estimates for radial basis function interpolation of scattered data. (English)

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Summary: Introducing a suitable variational formulation for the local error of scattered data interpolation by radial basis functions $\varphi(r)$, the error can be bounded by a term depending on the Fourier transform of the interpolated function f and a certain 'Kriging function', which allows a formulation as an integral involving the Fourier transform of φ. The explicit construction of locally well-behaving admissible coefficient vectors makes the Kriging function bounded by some power of the local density h of data points. This leads to error estimates for interpolation of functions f whose Fourier transform $\hat{f}$ is 'dominated' by the nonnegative Fourier transform $\hat{\psi}$ of $\psi(x) = \varphi(\|x\|)$ in the sense $\int |\hat{f}|^2 \hat{\psi}^{-1} dt < \infty$. Approximation orders are arbitrarily high for interpolation with Hardy multiquadrics, inverse multiquadrics and Gaussian kernels. This was also proven in recent papers by Madych and Nelson, using a reproducing kernel Hilbert space approach and requiring the same hypothesis as above on $\hat{f}$, which limits the practical applicability of the results. This work uses a different and simpler analytic technique and allows to handle the cases of interpolation with $\varphi(r) = r^s$ for $s \in \mathbb{R}$, $s > 1$, $s \notin 2\mathbb{N}$, and $\varphi(r) = r^s \log r$ for $s \in 2\mathbb{N}$, which are also to have accuracy $O(h^{s/2})$.

MSC:

41A05 Interpolation in approximation theory

Keywords:

local error of scattered data interpolation; radial basis functions; Fourier transform

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