Let $G$ be an algebraic group over algebraically closed field $K$, (with char $qK \geq 0$) and $V$ be a rational representation of $G$. In this detailed exposition the authors study the polystability which means that for the non-zero $v \in V$ its orbit $O_v$ is closed under the action of $G$ with the given representation.

In order to achieve the desired results, the authors use representation theoretic approach and use some important tools such as Grosshan’s principle, torus actions, Kempf’s theory. Sections 2, 3, and 4 are devoted for explaining these ideas and to some improvements.

In Section 5, the authors focus on representations of $SL(V)$ which is particular interest in invariant theory. In Section 6, closed orbits for cubic forms and tensor actions were studied in detail. In Section 7, the main results about the exponential lower degree bounds are proved.

Section 8 is devoted to the polystability of symmetric polynomials and the authors provide two algorithms; one for the case $p \nmid n$ and one for $p \mid n$.

Algorithm 8.11, $p \nmid n$:

Input. $f \in K[x_1, \ldots , x_n]_{d}$.

Step 1. Write $f = \sum_i t_i p_i$.

Step 2. If $l^{\lceil d/n \rceil +1} \mid f$ or $f = \sum_{i=0}^{\lceil d/n \rceil -1} l_i p_i$, then $f$ is unstable. Else, proceed to Step 3.

Step 3. If $n \nmid d$, then $f$ is polystable. Further, in this case, if $\dim SL(V)_f = 0$, then $f$ is stable. If $n \mid d$, proceed to Step 4.

Step 4. Check if $l^{d/n} \mid f$ or $f = \sum_{i=0}^{d/n} l_i p_i$. If neither holds, then $f$ is polystable. Further, in this case, if $\dim SL(V)_f = 0$, then $f$ is stable. If one $l^{d/n} \mid f$ or $f = \sum_{i=0}^{d/n} l_i p_i$ hold then go to Step 5.

Step 5. Let $f' = l^{d/n} f_{d/n}$. If $\dim SL(V)_{f'} = \dim SL(V)_f$, then $f$ is polystable and in this case if $\dim SL(V)_{f'} = 0$, then $f$ is stable. If $\dim SL(V)_{f'} \neq \dim SL(V)_f$, then $f$ is semistable, but not polystable.

Algorithm 8.13, $p \mid n$:

Input. $f \in K[x_1, \ldots , x_n]_d$.

Step 1. Compute $w = \text{ess}(f)$ as in Lemma 8.12. If $w = 0$, then $f$ is unstable. Else, proceed to Step 2.

Step 2. If $\dim SL(V)_f \neq \dim SL(V)_w$, then $f$ is semistable, not polystable. Else $f$ is polystable. Moreover, in the case that $f$ is polystable, $\dim SL(V)_f = 0$ if and only if $f$ is stable.

In the last section, the authors determine the polystability of some interesting symmetric polynomials, including; elementary, homogeneous and power sum symmetric polynomials, and Schur polynomials.

It has to be also noted that the motivation of the paper includes the relation of exponential degree bounds with the Geometric Complexity Theory (GCT) program which is an algebro-geometric approach to the celebrated $P$ vs $NP$ problem.

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MSC:

- 13A50 Actions of groups on commutative rings; invariant theory
- 05E10 Combinatorial aspects of representation theory
- 14L24 Geometric invariant theory
- 68W30 Symbolic computation and algebraic computation
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