Summary: We study a new class of NP search problems, those which can be proved total using standard combinatorial reasoning based on approximate counting. Our model for this kind of reasoning is the bounded arithmetic theory APC$_2$ of [E. Jeřábek, Approximate counting by hashing in bounded arithmetic, J. Symb. Log. 74(3) (2009) 829-860]. In particular, the Ramsey and weak pigeonhole search problems lie in the new class. We give a purely computational characterization of this class and show that, relative to an oracle, it does not contain the problem CPLS, a strengthening of PLS. As CPLS is provably total in the theory $T^2_2$, this shows that APC$_2$ does not prove every $\forall \Sigma^b_1$ sentence which is provable in bounded arithmetic. This answers the question posed in [S. Buss, L. A. Kołodziejczyk and N. Thapen, Fragments of approximate counting, J. Symb. Log. 79(2) (2014) 496-525] and represents some progress in the program of separating the levels of the bounded arithmetic hierarchy by low-complexity sentences. Our main technical tool is an extension of the “fixing lemma” from [P. Pudlák and N. Thapen, Random resolution refutations, Comput. Complexity, 28(2) (2019) 185-239], a form of switching lemma, which we use to show that a random partial oracle from a certain distribution will, with high probability, determine an entire computation of a $\mathcal{P}^{\mathcal{NP}}$ oracle machine. The introduction to the paper is intended to make the statements and context of the results accessible to someone unfamiliar with NP search problems or with bounded arithmetic.

MSC:
03F30 First-order arithmetic and fragments
03D15 Complexity of computation (including implicit computational complexity)
68Q15 Complexity classes (hierarchies, relations among complexity classes, etc.)
68Q17 Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.)
03F20 Complexity of proofs

Keywords:
bounded arithmetic; NP search problems; approximate counting; weak pigeonhole principle; CPLS; switching lemmas

Full Text: DOI arXiv

References:
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