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Polynomial size constant depth circuits with a limited number of negations. (English)

Zbl 0764.94025
Theoretical aspects of computer science, Proc. 8th Annu. Symp., STACS '91, Hamburg/Ger. 1991, Lect. Notes Comput. Sci. 480, 228-237 (1991).

Summary: [For the entire collection see Zbl 0753.00019.]

It follows from a theorem of Markov that the minimum number of negation gates in a circuit sufficient to compute any Boolean function on n variables is $l = \lfloor \log n \rfloor + 1$. It can be shown that, for functions computed by families of polynomial size, $O(\log n)$ depth and bounded fan-in circuits (NC^1), the same result holds: on such circuits l negations are necessary and sufficient. In this paper we prove that this situation changes when polynomial size circuit families of constant depth are considered: l negations are no longer sufficient. For threshold circuits we prove that there are Boolean functions computable in constant depth (TC^0) such that no such threshold circuit containing $o(n^\varepsilon)$, for all $\varepsilon > 0$, negations can compute them. We have a matching upper bound: for any $\varepsilon > 0$, everything computed by constant depth threshold circuits can be so computed using n^ε negations asymptotically. We also have tight bounds for constant depth, unbounded fan-in circuits (AC^0) : $n/\log^r n$, for any r, negations are sufficient, and $\Omega(n/\log^r n)$, for some r, are necessary.

MSC:

94C10 Switching theory, application of Boolean algebra; Boolean functions (MSC2010)

Keywords:

circuit; Boolean function; fan-in circuits; threshold circuits