Calderón-Villalobos, Angel; Sánchez, Iván

Hattori topologies on almost topological groups. (English) Zbl 07653711
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Authors’ abstract: For a subset $A$ of an almost topological group $G$, we define the Hattori topological space $H(A)$, where $H(A)$ is a topological space whose underlying set is $G$ and whose topology is defined as follows: if $x \in A$ (respectively, $x \notin A$), then the neighborhoods of $x$ in $H(A)$ are the same neighborhoods of $x$ in the reflection group (respectively, $G$). In this paper, we show that if $G$ is an almost topological group and $A$ is a proper subset of $G$, then $H(A)$ is regular if and only if $G$ is regular. We also prove that $\chi(H(A)) = \chi(G)$ for each proper subset $A$ of $G$. If $G$ is an almost topological group and $G$ is not a topological group, we show the following:

i) For each infinite subspace $B$ of $G$, we have that $n\omega(B) = |B|$.

ii) If $A$ is a proper subset of $G$, then $\omega(H(A)) = d(G) \cdot \chi(G) \cdot |G \setminus A|$.

iii) In particular, if $A$ is a proper subset of $G$, then $H(A)$ is second-countable if and only if $G$ is first-countable separable and $G \setminus A$ is countable.

iv) If $A$ is a subset of $G$, then $n\omega(H(A)) = n\omega(G) \cdot (|G \setminus A| + \omega)$.

Reviewer: Zhangyong Cai (Nanning)

MSC:
54H11 Topological groups (topological aspects)
54A25 Cardinality properties (cardinal functions and inequalities, discrete subsets)
54A10 Several topologies on one set (change of topology, comparison of topologies, lattices of topologies)

Keywords:
almost topological groups; paratopological groups; Hattori spaces; reflection group; second-countable; first-countable; network weight

Full Text: DOI

References:
[13] Lin, F.; Li, J., Some topological properties of spaces between the Sorgenfrey and usual topologies on real number

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