
Let $C(X), \Gamma(X)$ denote the closed nonempty convex subsets of a Banach space $X$ and the proper lower semicontinuous convex functions on $X$. $C(X)$ is equipped with the topology $\tau$ of uniform convergence of distance functions on bounded subsets of $X$ and on $\Gamma(X)$ the related topology is defined: $\lim f_n = f$ iff $\lim (\text{epi} f_n) = \text{epi} f$ in $(C(X), \tau)$. It is proved that under standard regularity assumptions operations of addition and restriction are continuous on $(\Gamma(X), \tau)$. These results are applied to convex well-posed optimization problems

$$\min \{f(x) | x \in A\} \quad (= v(f, A)); \quad f \in \Gamma(X), \quad A \in C(X). \quad (1)$$

Well-posedness of (1) means that relations $\lim_{n \to \infty} f(x_n) = v(f|A), \ x_n \in A$ imply $\lim_{n \to \infty} x_n = x_*$ where $\{x_*\} = \text{Argmin}\{f(x) | x \in A\}$.

The following theorem is proved: Let $\{(f_n, A_n)\}$ be a sequence in $(\Gamma(X), \tau) \times (C(X), \tau)$ convergent to $(f, A)$. Suppose the problem (1) is well-posed and either $f$ is continuous at some point of $A$ or $\text{dom} f \cap \text{int} A \neq \emptyset$. Then (a) $v(f|A) = \lim_{n \to \infty} v(f_n|A_n)$; (b) if $f_n(x_n) < v(f_n|A_n) + \varepsilon_n, \ x_n \in A_n, \ \varepsilon_n > 0$, $\lim_{n \to \infty} \varepsilon_n = 0$ then $\lim_{n \to \infty} x_n = x_*$. It is also shown that in a natural sense for most $(f, A)$ in $(\Gamma(X), \tau) \times (C(X), \tau)$ the function $f$ is continuous and whenever $\limsup_{n \to \infty} f(x_n) \leq v(f, A), \ \lim_{n \to \infty} d(x_n, A) = 0$ then $\lim_{n \to \infty} x_n = x_*.$

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