Summary: In the present paper, we introduce two-dimensional categorified Hall algebras of smooth curves and smooth surfaces. A categorified Hall algebra is an associative monoidal structure on the stable ∞-category $\text{Coh}^b(\mathcal{R}\mathcal{M})$ of complexes of sheaves with bounded coherent cohomology on a derived moduli stack $\mathcal{R}\mathcal{M}$. In the surface case, $\mathcal{R}\mathcal{M}$ is a suitable derived enhancement of the moduli stack $\mathcal{M}$ of coherent sheaves on the surface. This construction categorifies the K-theoretical and cohomological Hall algebras of coherent sheaves on a surface of Zhao and Kapranov-Vasserot. In the curve case, we define three categorified Hall algebras associated with suitable derived enhancements of the moduli stack of Higgs sheaves on a curve $X$, the moduli stack of vector bundles with flat connections on $X$, and the moduli stack of finite-dimensional local systems on $X$, respectively. In the Higgs sheaves case we obtain a categorification of the K-theoretical and cohomological Hall algebras of Higgs sheaves on a curve of Minets and Sala-Schiffmann, while in the other two cases our construction yields, by passing to $K_0$, new K-theoretical Hall algebras, and by passing to $H_{stBM}$, new cohomological Hall algebras. Finally, we show that the Riemann-Hilbert and the non-abelian Hodge correspondences can be lifted to the level of our categorified Hall algebras of a curve.


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