Lin, Fucai; Wu, Qiyun

The characterizations of dense-pseudocompact and dense-connected spaces. (English)

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Summary: Assume that $\mathcal{P}$ is a topological property of a space $X$, then we say that $X$ is dense-$\mathcal{P}$ if each dense subset of $X$ has the property $\mathcal{P}$. In this paper, we mainly discuss dense subsets of a space $X$, and we prove that:

1. if $X$ is Tychonoff space, then $X$ is dense-pseudocompact if and only if the range of each continuous real-valued function $f$ on $X$ is finite, if and only if $X$ is finite, if and only if $X$ is hereditarily pseudocompact;

2. $X$ is dense-connected if and only if $U = X$ for any non-empty open subset $U$ of $X$;

3. $X$ is dense-ultraconnected if and only if for point $x \in X$, we have $\{x\} = X$ or $\{x\} \cup (X \setminus \{x\})$ is the unique open neighborhood of $x$ in $\{x\} \cup (X \setminus \{x\})$, if and only if for any two points $x$ and $y$ in $X$, we have $x \in \{y\}$ or $y \in \{x\}$.

Moreover, we give a characterization of a topological group (resp., paratopological group, quasi-topological group) $G$ such that $G$ is dense-connected.

MSC:

- 22A05 Structure of general topological groups
- 54B05 Subspaces in general topology
- 54C30 Real-valued functions in general topology
- 54D05 Connected and locally connected spaces (general aspects)
- 54H11 Topological groups (topological aspects)

Keywords:

dense-pseudocompact; dense-connected; dense-subgroup-connected; dense-ultraconnected; dense subset

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References:


Lin, F.; Wu, Q. Y.; Liu, C., Dense-separable groups and its applications in d-independence


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