Sattinger, D. H.; Szmigielski, J. S.

Factorization and the dressing method for the Gel'fand-Dikii hierarchy. (English)

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Summary: The isospectral flows of an nth order linear scalar differential operator L under the hypothesis that it possesses a Baker-Akhiezer function were originally investigated by Segal and Wilson from the point of view of infinite dimensional Grassmannians, and the reduction of the KP hierarchy to the Gel'fand-Dikii hierarchy. The associated first order system and their formal asymptotic solutions have a rich Lie algebraic structure which was investigated by Drinfeld and Sokolov. We investigate the matrix Riemann-Hilbert factorizations for these systems, and show that different factorizations lead respectively to the potential, modified, and ordinary Gel'fand-Dikii flows. Lie algebra decompositions (the Adler-Kostant-Symes method) are obtained for the modified and potential flows. For $n > 3$ the appropriate factorization for the Gel'fand-Dikii flows is not a group factorization, as would be expected; yet a modification of the dressing method still works. A direct proof, based on a Fredholm determinant associated with the factorization problem, is given that the potentials are meromorphic in x and in the time variables. Potentials with Baker-Akhiezer functions include the multisoliton and rational solutions, as well as potentials in the scattering class with compactly supported scattering data. The latter are dense in the scattering class.

MSC:

34L40 Particular ordinary differential operators (Dirac, one-dimensional Schrödinger, etc.)

34C20 Transformation and reduction of ordinary differential equations and systems, normal forms

Keywords:

isospectral flows; nth order linear scalar differential operator; Baker-Akhiezer function; matrix Riemann-Hilbert factorizations; Gel'fand- Dikii flows; Lie algebra decompositions; Adler-Kostant-Symes method; potential flows; dressing method; scattering class

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