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Space-time spectral method for the Stokes problem. (English) Zbl 07705772


Summary: The Stokes equations are a linearized version of the Navier-Stokes equations and model incompressible viscous fluid flow with low Reynolds numbers. Several spectral methods, exhibiting exponential decay in error when the solution is analytic, are known to solve the steady-state Stokes problem numerically. A common strategy to solve such a problem in the time-dependent case involves extending the spectral scheme in spatial derivatives by implementing a low-order finite difference scheme for the time derivatives. Instead, we implement and analyze a space-time spectral method for the Stokes problem, which converges exponentially in both space and time. This numerical scheme imposes spectral collocation in time and $P_N - P_{N-2}$ spectral Galerkin scheme in space by using a recombined Legendre polynomial basis, resulting in a global spectral operator that is a saddle point matrix. The main objectives of the research are estimating the condition number of the global spectral operators and proving the spectral convergence of this scheme in space and time. The analysis is not quite complete because two of the estimates are based on numerical evidence. However, throughout the project, some intermediate results—such as the 2-norm of the pseudospectral derivative matrix for Chebyshev-Gauss-Lobatto nodes as well as the condition number of the mass matrix and discrete Laplacian for a recombined Legendre basis—were proved to obtain the aforementioned findings. Numerical experiments of this scheme verify the theoretical results. Also, the numerical result of applying this scheme to the Navier-Stokes equations is presented.

MSC:

- 65Mxx Numerical methods for partial differential equations, initial value and time-dependent initial-boundary value problems
- 65Nxx Numerical methods for partial differential equations, boundary value problems
- 15Axx Basic linear algebra

Keywords:
- condition number; recombined Legendre polynomials; space-time; spectral collocation; spectral Galerkin; Stokes problem

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References:


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