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Periodic and homoclinic solutions to a class of Hamiltonian systems with the potentials changing sign. (English) Zbl 0771.34031

Dyn. Syst. Appl. 2, No. 1, 131-145 (1993).

This paper considers the Hamiltonian system $(*)$ $\ddot{q}=B(t)q+b(t)W'(q)=0$ $(q\in R^N)$ with the following assumptions: (V_1) B is a continuous, T-periodic, positive definite and symmetric matrix valued function; (V_2) b is a continuous T-periodic real function, and there exist t_1, t_2 such that $b(t_1)>0$ and $b(t_2)<0$; (V_3) $W\in C'(R^N,R)$, $W(x)>0$ $\forall x\neq 0$ and there exists $\mu>2$ such that $W(\lambda x)=\lambda^\mu W(x)$ $\forall\lambda\geq 0$ and $x\in R^N$. The main results consist of the following theorems: 1) Assume $(\mathrm{V}_1)-(\mathrm{V}_2)-(\mathrm{V}_3)$. Then $(*)$ has at least one nonconstant T-periodic solution; 2) Assume $(\mathrm{V}_1)-(\mathrm{V}_2)-(\mathrm{V}_3)$, and moreover, $W\in C^2(R^N,R)$ is even. Then $(*)$ has infinitely many (pairs of) nonconstant T- periodic solutions; 3) Assume $(\mathrm{V}_1)-(\mathrm{V}_2)-(\mathrm{V}_3)$. Then $(*)$ possesses a homoclinic solution $q(t)$, emanating from zero such that $q\in W^{1,2}(R^N,R)$.

Reviewer: Ding Tongren (Beijing)

MSC:

34C25 Periodic solutions to ordinary differential equations

34C37 Homoclinic and heteroclinic solutions to ordinary differential equations

Keywords:

periodic solution; Hamiltonian system; homoclinic solution