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Identities for combinatorial sums involving trigonometric functions. (English) Zbl 07714098

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Summary: Let

\[ A_{m,n}(a) = \sum_{j=0}^{m} (-4)^j \left( \frac{m+j}{2j} \right) \sum_{k=0}^{n-1} \sin\left(a + \frac{2k\pi}{n}\right) \cos^{2j}(a + \frac{2k\pi}{n}) \]

and

\[ B_{m,n}(a) = \sum_{j=0}^{m} (-4)^j \left( \frac{m+j+1}{2j+1} \right) \sum_{k=0}^{n-1} \sin\left(a + \frac{2k\pi}{n}\right) \times \cos^{2j+1}(a + \frac{2k\pi}{n}), \]

where \( m \geq 0 \) and \( n \geq 1 \) are integers and \( a \) is a real number. We present two proofs for the following results:

(i) If \( 2m + 1 \equiv 0 \pmod{n} \), then

\[ A_{m,n}(a) = (-1)^m n \sin((2m+1)a). \]

(ii) If \( 2m + 1 \not\equiv 0 \pmod{n} \), then \( A_{m,n}(a) = 0. \)

(iii) If \( 2(m+1) \equiv 0 \pmod{n} \), then

\[ B_{m,n}(a) = (-1)^m \frac{n}{2} \sin(2(m+1)a). \]

(iv) If \( 2(m+1) \not\equiv 0 \pmod{n} \), then \( B_{m,n}(a) = 0. \)

MSC:

05A19 Combiantorial identities, bijective combinatorics
33B10 Exponential and trigonometric functions
33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

Keywords:

combinatorial identity; trigonometric function; Chebyshev polynomials of the first and second kind

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References:


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