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Gröbner bases: a computational approach to commutative algebra. In cooperation with Heinz Kredel. (English) [Zbl 0772.13010](#)

Graduate Texts in Mathematics. 141. New York: Springer-Verlag. xxii, 574 p. (1993).

A Gröbner basis G of an ideal I of a polynomial ring $K[x_1, \dots, x_n]$, K a field, is a special set of generators that leads to solutions of many algorithmic problems in the theory of polynomial ideals. If such an ideal is given by a finite set of generators the Buchberger algorithm (that is now implemented in each computer algebra system) produces a Gröbner basis of this ideal. — The authors present an introduction into the theory and applications of Gröbner bases that “requires no prerequisites other than the mathematical maturity of an advanced undergraduate student” (from the preface). So this book includes an introduction to commutative algebra.

The first three chapters contain the necessary material from the theory of rings and modules. Chapter 4 treats of abstract reduction relations in ordered monoids. In chapter 5 the existence and uniqueness of Gröbner bases is proved and Buchberger’s algorithm is described. Chapters 6 to 9 concern basic constructions in ideal theory, Hilbert’s Nullstellensatz, primary decomposition, the Hilbert polynomial, and the usefulnesses of Gröbner bases for all these things. In addition, Sturm’s isolation method for real zeros of polynomials is presented. Chapter 10 contains “variations”: more general coefficient rings, homogenization, Gröbner bases of modules, standard bases. – An appendix treats with advanced topics: the complexity of Gröbner basis constructions, Gröbner bases with parameters, automatic theorem proving, non-commutative rings. A “selected bibliography” of 28 pages and a list of symbols close the book.

All explanations are very detailed and clear, nearly 50 algorithms are presented in an appropriate formal way. The authors recommend the use of the computer algebra system MAS by H. Kredel.

Reviewer: [H.Grassmann \(Berlin\)](#)

MSC:

- [13P10](#) Gröbner bases; other bases for ideals and modules (e.g., Janet and border bases)
- [13-01](#) Introductory exposition (textbooks, tutorial papers, etc.) pertaining to commutative algebra
- [65H99](#) Nonlinear algebraic or transcendental equations

Cited in 1 Review Cited in 203 Documents

Keywords:

[Gröbner basis](#); [Buchberger algorithm](#)