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Axiomatizability and completeness of some classes of S-polygons. (English. Russian original)

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Let K be a class of algebraic systems of a signature Σ. Recall that a class K is said to be axiomatizable if there exists a set Z of propositions of signature Σ such that the class K contains precisely those systems on which are propositions in Z are true. A class K is said to be complete (model complete) if the theory of the class K_∞ of all infinite systems in K is complete (model complete). A class K is said to be categorical if it is categorical in some uncountable cardinality.

Let S be a monoid; let K be the class of flat S-polygons, or the class of projective S-polygons, or the class of free S-polygons; let a property P of the class K be axiomatizability, or completeness, or model completeness, or categoricity. It is natural to raise this qestion: What conditions should the monoid S satisfy in order for the class K to possess the property P? Necessary and sufficient conditions that one has to impose on a monoid S in order for the class of flat S-polygons to be axiomatizable are stated by *V. Gould* [J. Lond. Math. Soc., II. Ser. 35, 193-201 (1987; Zbl 0637.03029)]. That paper also proves that for a monoid S satisfying the ascending chain condition M^L for principal left ideals the axiomatizability of the class of projective S-polygons is equivalent to the axiomatizability of the class of flat S-polygons and to the monoid S being perfect. Theorem 1 of the present article generalizes this result to the case of an arbitrary monoid. Theorem 2 provides a description of a monoid $S$with finitely many right ideals for which the class of free S-polygons is axiomatizable. Theorems 3 and 4 prove that for a (commutative) monoid S, completeness, model completeness, and categoricity of the class $\mathcal{P}(\mathcal{F})$ are equivalent to S being a group.

MSC:

03C60 Model-theoretic algebra
08C10 Axiomatic model classes

Keywords:

monoid; flat S-polygons; projective S-polygons; free S-polygons; axiomatizability; completeness; model completeness; categoricity; ascending chain condition; ideals

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