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A new second order Taylor-like theorem with an optimized reduced remainder.  (English)

Zbl 0775.6726

Summary: In this paper, we derive a variant of the Taylor theorem to obtain a new minimized remainder. For a given function \( f \) defined on the interval \([a, b]\), this formula is derived by introducing a linear combination of \( f' \) computed at \( n+1 \) equally spaced points in \([a, b]\), together with \( f''(a) \) and \( f''(b) \). We then consider two classical applications of this Taylor-like expansion: the interpolation error and the numerical quadrature formula. We show that using this approach improves both the Lagrange \( P_2 \)-interpolation error estimate and the error bound of the Simpson rule in numerical integration.

MSC:

26Dxx  Inequalities in real analysis
65Dxx  Numerical approximation and computational geometry (primarily algorithms)
41Axx  Approximations and expansions

Keywords:

Taylor’s theorem; Lagrange interpolation; interpolation error; Simpson rule; quadrature error

Full Text: DOI  arXiv

References:

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