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Radicals of H-module algebras. (English. Russian original) Zbl 0776.16013

Algebra Logic 28, No. 6, 462-474 (1989); translation from Algebra Logika 28, No. 6, 705-721 (1989).

Let k be a fundamental field. An associative k-algebra H, with identity element 1, is said to be a Hopf algebra if on it there are defined algebra homomorphisms $\Delta : H \to H \otimes H$, $\varepsilon : H \to k$, and a mapping $s : H \to H$, satisfying the following conditions:

$$\text{if } \Delta h = \sum_{(h)} h_{(1)} \otimes h_{(2)} \text{ then } \sum_{(h)} \Delta h_{(1)} \otimes h_{(2)} = \sum_{(h)} h_{(1)} \otimes \Delta h_{(2)},$$

$$\sum_{(h)} \varepsilon(h_{(1)})h_{(2)} = \sum_{(h)} h_{(1)} \cdot \varepsilon(h_{(2)}) = h, \quad \sum_{(h)} s(h_{(1)})h_{(2)} = \sum_{(h)} h_{(1)} s(h_{(2)}) = \varepsilon(.) \cdot 1.$$

An associative algebra A is said to be an H-module algebra if A is a unital right H-module (where the action of H on A is written in the form a^h) and we have the equality $(ab)^h = \sum_{(h)} a^{h_{(1)}} b^{h_{(2)}}$ for all $a, b \in A$, $h \in H$. If A has an identity, then one imposes additionally the condition $1^h = \varepsilon(h) \cdot 1$ for all $h \in H$.

The author studies the theory of classical H-radicals of associative algebras. The Anderson-Divinsky-Sulinski Theorem (ADS-Theorem) states that if A is an associative algebra, $B \lhd A$, and ρ is an arbitrary radical, then $\rho(B) \lhd A$. Those H- radicals for which the ADS-Theorem holds will be called ADS-radicals. It is known that each hypernilpotent H-radical is an ADS-radical. The author shows that if H is a finite dimensional semisimple Hopf algebra, different from the fundamental field k, then the ADS-Theorem does not hold for H-radicals. The author then shows that the theory of nil-H- radicals is basically similar to the classical one, while the Jacobson radical generates an entire spectrum of different H-radicals. In the final section of the paper, the author defines and studies strongly H- semisimple algebras.

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MSC:

16S90	Torsion theories; radicals on module categories (associative algebraic aspects)
16N20	Jacobson radical, quasimultiplication
16N40	Nil and nilpotent radicals, sets, ideals, associative rings
16W30	Hopf algebras (associative rings and algebras) (MSC2000)
16N60	Prime and semiprime associative rings
16S40	Smash products of general Hopf actions

Keywords:

algebras with Hopf algebra actions; Hopf algebra; algebra homomorphisms; H-module algebra; Anderson-Divinsky-Sulinski theorem; ADS-radicals; hypernilpotent H-radical; finite dimensional semisimple Hopf algebra; nil-H-radicals; Jacobson radical; strongly H-semisimple algebras

Full Text: DOI

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