Sergeev, A. G.

On matrix Reinhardt and circled domains. (English) Zbl 0778.32002

Several complex variables, Proc. Mittag-Leffler Inst., Stockholm/Swed. 1987-88, Math. Notes 38, 573-586 (1993).

[For the entire collection see Zbl 0759.00008.]

The main purpose of this paper is to give a criterion for holomorphic convexity of matrix Reinhardt domains.

A domain D in the space of n matrix variables with $m \times m$ entries is said to be Reinhardt if for each point $(Z_1, \ldots, Z_n) \in D$, the point $(U_1 Z_1 V_1, \ldots, U_n Z_n V_n)$ also lies in D for all unitary $m \times m$-matrices U_j, V_k. If $(Z_1, \ldots, Z_n) \in D \Rightarrow (UZ_1V, \ldots, UZ_nV) \in D$ for all unitary $m \times m$ matrices U, V, D is said to be circled. Several examples of matrix Reinhardt and circled domains are given in the paper.

Given a matrix Reinhardt domain D, set $\operatorname{diag} D = \{(\Lambda_1, \ldots, \Lambda_n) \in D : \Lambda_i$ is a diagonal matrix, $1 \le i \le n\}$; diag D is a (scalar) Reinhardt open set in the space $\mathbb{C}^{mn}$.

Theorem: Let D be a complete matrix Reinhardt domain. Then D is holomorphically convex if and only if diag D is holomorphically convex.

Here completeness means that for each point $(Z_1^0, \ldots, Z_n^0) \in D$, the matrix polydisk

$$\{(Z_1, \ldots, Z_n) : \|Z_i\| \le \|Z_i^0\|, \qquad 1 \le i \le n\}$$

with respect to the spectral matrix norm $\|Z\| = \max$ {eigenvalues of $\sqrt{Z * Z}$} also lies in D.

Since, with the polar representation of a matrix in mind, it is natural to define the logarithmic image of D as the logarithmic image of diag D, the author's theorem can be considered as matrix analogue of the well-known criterion for holomorphic convexity of Reinhardt (scalar) domains. A similar result was proved independently by *G. Khudajberganov* [Mat. Vesn. 40, No. 3/4, 241-248 (1988; Zbl 0702.32001)].

Reviewer: J.Davidov (Sofia)

MSC:

32A07 Special domains in $\mathbb{C}^n$ (Reinhardt, Hartogs, circular, tube) (MSC2010)
32D05 Domains of holomorphy

Keywords:

circled domain; holomorphic convexity; matrix Reinhardt domains