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Two timescale harmonic balance. I: Application to autonomous one-dimensional nonlinear oscillators. (English) [Zbl 0778.34022](#)

Philos. Trans. R. Soc. Lond., Ser. A 340, No. 1659, 473-501 (1992).

Two timescale harmonic balance is a semi-analytic/numerical method for deriving periodic solutions and establishing their stability. In this first paper the method is applied to a class of nonlinear autonomous oscillators which can be described by differential equations of the type $\ddot{x} + x = f(x, \dot{x}, \lambda, t)$, where λ is a control parameter. Features of both harmonic balance and multiple scales are incorporated in the method. The solution $x(t)$ is sought as a series of superharmonics, subharmonics and ultrasubharmonics. The two timescales, associated with the amplitude and phase variations, are introduced through a parameter ε . The method is applied to three versions of the van der Pol equation. Expansions in superharmonics reveal Hopf, saddle-node and homoclinic bifurcations.

Reviewer: P.Smith (Keele)

MSC:

[34C15](#) Nonlinear oscillations and coupled oscillators for ordinary differential equations

Cited in **3** Documents

[34C23](#) Bifurcation theory for ordinary differential equations

[65J99](#) Numerical analysis in abstract spaces

Keywords:

two timescale harmonic balance; series of ultrasubharmonics; periodic solutions; stability; nonlinear autonomous oscillators; multiple scales; series of superharmonics; series of subharmonics; van der Pol equation; Hopf bifurcations; saddle-node and homoclinic bifurcations

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