On the fractional matching polytope of a hypergraph. (English) Zbl 0779.05030

Let $\mathcal{H} = (V, E)$ be a hypergraph where $V$ is a finite set and $E$ is a multiset of subsets of $V$. A subset $\mathcal{M}$ of $E$ is called a matching if every two of its members are disjoint, and a function $w : E \to \mathbb{R}_+$ is called a fractional matching if $\sum_{e \in E} w(e) \leq 1$ for all $v \in V$. For $b : E \to \mathbb{R}_+$ let

$$\nu_b = \max \left\{ \sum_{e \in \mathcal{M}} b(e) : \mathcal{M} \text{ is a matching} \right\},$$

$$\nu^*_b = \max \left\{ \sum_{e \in E} b(e)w(e) : w \text{ is a fractional matching} \right\}.$$

In the case $b \equiv 1$ write briefly $\nu$ and $\nu^*$. $\mathcal{H}$ is called $k$-uniform if $|e| = k$ for all $e \in E$, and $\mathcal{H}$ is called intersecting if $\nu = 1$. The following theorems are proved:

(1) Any hypergraph $\mathcal{H}$ has a matching $\mathcal{M}$ with $\sum_{e \in \mathcal{M}} (|e| - 1 + \frac{1}{|e|}) \geq \nu^*$.

(2) For any $k$-uniform hypergraph $\mathcal{H}$ and any $b : E \to \mathbb{R}_+$ we have $(k - 1 + \frac{1}{k}) \nu_b \geq \nu^*_b$.

(3) If $w$ is a fractional matching of an intersecting hypergraph $\mathcal{H}$, then $\sum_{e \in E} w(e) |e|^{-1+1/|e|} \leq 1$.

(4) If $\mathcal{H}$ is $k$-uniform and intersecting, and $\bigcap_{e \in \mathcal{H}} e = \emptyset$, then $\frac{1}{|\mathcal{E}|^2} \sum_{e \in E} \sum_{f \in E} |e \cap f| \geq \frac{k^2}{k^2-k+1}$.

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MSC:
05C65 Hypergraphs
05C70 Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)
05D15 Transversal (matching) theory

Keywords:
fractional matching polytope; $k$-uniform hypergraph; hypergraph; matching; fractional matching; intersecting hypergraph

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References:

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