Bovdi, A. A.; **Sehgal, S. K.**

Unitary subgroup of integral group rings. (English) Zbl 0779.16014

Manuscr. Math. 76, No. 2, 213-222 (1992).

Let $\mathbb{Z}G$ be the integral group ring of an arbitrary group G and $U(\mathbb{Z}G)$ be the group of units of $\mathbb{Z}G$. Let $f : G \to U(\mathbb{Z})$ be an orientation homomorphism of the group G, and for each $x = \sum_{g\in G} a_g g$ in $\mathbb{Z}G$ set $x^f = \sum_{g\in G} a_g f(g)g^{-1}$. The collection of elements $U_f(\mathbb{Z}G) \equiv \{u \in U(\mathbb{Z}G) \mid u^{-1} = u^f \text{ or } u^{-1} = -u^f\}$ forms a subgroup of $U(\mathbb{Z}G)$. This subgroup has been investigated by several authors. If $U_f(\mathbb{Z}G) = U(\mathbb{Z}G)$, then $U(\mathbb{Z}G)$ is called f-unitary. By the first author [in Mat. Sb., Nov. Ser. 119, No. 3, 387-400 (1982; Zbl 0511.16009)] necessary conditions for $U(\mathbb{Z}G)$ to be f-unitary were given and many of them were shown to be sufficient. In the present paper, the authors' main result is on the problem of normality of $U_f(\mathbb{Z}G)$ in $U(\mathbb{Z}G)$. They give necessary conditions for normality. These conditions are sufficient in many cases and they imply easily the necessary conditions for $U(\mathbb{Z}G)$ to be f-unitary. One of the sufficiency cases in the above mentioned paper for f-unitarity of $U(\mathbb{Z}G)$ is improved while another case still remains open.

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MSC:

16U60	Units, groups of units (associative rings and algebras)
20C07	Group rings of infinite groups and their modules (group-theoretic aspects)
16S34	Group rings

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integral group ring; group of units; conditions for normality; f- unitarity

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