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Nonpositively curved Kähler metrics on product manifolds. (English) Zbl 0779.53045

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The main purpose of this note is to study a metric with certain curvature conditions on the product of two compact complex manifolds $M = M_1 \times M_2$. Let $\mathcal{F}(M)$ denote the space of all those Kähler metrics on M with nonpositive bisectional curvature. Let π_i be the projection map from M to M_i with $q_i = h^{1,0}(M_i)$, $i = 1, 2$. Let $\{\varphi_1, \dots, \varphi_{q_1}\}$ and $\{\psi_1, \dots, \psi_{q_2}\}$ be the basis of global holomorphic 1 forms on M_1 and M_2 respectively, and ω_g the Kähler form of a metric g. If $\mathcal{F}(M) \neq \emptyset$ and $g \in \mathcal{F}(M)$ the author proves that there exist $g_i \in \mathcal{F}(M_i)$ and a constant $q_1 \times q_2$-matrix (a_{ij}) such that

$$\omega_g = \pi_1^* \omega_{g_1} + \pi_2^* \omega_{g_2} + \rho + \bar{\rho},$$

where $\rho = \sum_{i=1}^{q_1} \sum_{j=1}^{q_2} a_{ij} \varphi_i \wedge \bar{\psi}_j$. Furthermore, for any $g_i \in \mathcal{F}(M_i)$ and any global holomorphic 1-forms $\eta_1, \dots, \eta_r$ on M, $\omega = \pi_1^* \omega_1 + \pi_2^* \omega_2 + \sqrt{-1} \sum_{i=1}^{r} \eta_i \wedge \bar{\eta}_i$ is the Kähler form of some $g \in \mathcal{F}(M)$. Therefore

$$\operatorname{codim}_{\mathbb{R}}(\mathcal{F}(M_1) \times \mathcal{F}(M_2);\ \mathcal{F}(M)) = \frac{1}{2} \cdot b_1(M_1) \cdot b_1(M_2).$$

In particular $\mathcal{F}(M) = \mathcal{F}(M_1) \times \mathcal{F}(M_2)$ if and only if $b_1(M_1) = 0$ or $b_1(M_2) = 0$.

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bisectional curvature; holomorphic 1 forms; Kähler form

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