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Nearly ordinary Hecke algebras and Galois representations of several variables. (English)

Zbl 0782.11017
Algebraic analysis, geometry, and number theory, Proc. JAMI Inaugur. Conf., Baltimore/MD (USA) 1988, 115-134 (1989).

[For the entire collection see Zbl 0747.00038.]

The purpose of this paper is to supplement the author's previous papers on Hecke algebras over totally real fields with a result on the canonical Galois representations into GL_2 with coefficients in the total quotient rings of the Hecke algebras.

More specifically the author proves the following two theorems: Let S be an open compact subgroup of $GL_2(\prod_{\mathfrak{p}} \mathfrak{r}_{\mathfrak{p}})$ containing $U_1(N)$ with $\mathfrak{r}_{\mathfrak{p}}$ the completion at a prime ideal $\mathfrak{p}$ of a totally real field F of finite degree, and $\mathcal{O}$ the p-adic integer ring of a finite extension of the closure of the field generated by all the conjugates of F. $\mathbf{h}(S;\mathcal{O})$ denotes the full Hecke algebra of infinite p-power level and $\mathbf{h}^{n,\mathrm{ord}}(S;\mathcal{O})$ is the nearly ordinary part. Then:

Theorem 1. Let A be an integral domain of characteristic different from 2 and $\lambda : \mathbf{h}^{n,\mathrm{ord}}(S;\mathcal{O}) \to A$ be a continuous $\mathcal{O}$-algebra homomorphism. Let $\mathcal{Q}$ be the quotient field of A. Then there exists a unique semisimple Galois representation $\pi : Gal(\mathbb{Q}/F) \to GL_2(\mathcal{Q})$ such that: (i) π is continuous; (ii) π is unramified outside Np, where N is the level of S; (iii) For the Frobenius element $\varphi_{\mathfrak{q}}$ for each prime $\mathfrak{q}$ outside Np,

$$\det(1-\pi(\varphi_{\mathfrak{q}})X) = 1 - \lambda(T(\mathfrak{q}))X + \lambda(\langle\mathfrak{q}\rangle)\mathfrak{N}_{F/\mathbb{Q}}(\mathfrak{q})X^2;$$

(iv) Let $\mathfrak{p}$ be a prime factor of p and fix a decomposition group $D_{\mathfrak{p}}$ in $Gal(\overline{\mathbb{Q}}/F)$. Then there exist two characters ε, δ of $D_{\mathfrak{p}}$ with values in A such that the restriction of π to $D_{\mathfrak{p}}$ is, up to equivalence, of the following form:

$$\pi(\sigma) = \begin{pmatrix} \varepsilon(\sigma) & * \\ 0 & \delta(\sigma) \end{pmatrix} \qquad \text{for } \sigma \in D_{\mathfrak{p}}.$$

Moreover if A and λ satisfy additional conditions which are given specifically in the paper, π is absolutely irreducible.

Theorem 2 is the similar statement to Theorem 1 without (iv) by replacing $\lambda : \mathbf{h}^{n,\mathrm{ord}}(S;\mathcal{O}) \to A$ by $\lambda : \mathbf{h} \to A$. From this one can associate a canonical Galois representation to any p-adic common eigenform of all Hecke operators.

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MSC:

11F85 p-adic theory, local fields
11F80 Galois representations
11S23 Integral representations

Keywords:

p-adic modular form; Hecke algebras over totally real fields; Galois representations