Lowen, R.
Cantor-connectedness revisited. (English) Zbl 0782.54010
Commentat. Math. Univ. Carol. 33, No. 3, 525-532 (1992).

In [Math. Nachr. 141, 183-226 (1989; Zbl 0676.54012)] the author has introduced approach spaces as a common generalization of topological spaces and of metric spaces. In particular the category $\mathbf{A}P$ of approach spaces is a topological construct that contains (a) the category **Top** of topological spaces and continuous maps as a simultaneously bireflective and bicoreflective full (!) subcategory and (b) the category **PMet** of pseudometric spaces and nonexpansive maps as a bicoreflective full (!) subcategory. In the present paper the author singles out a class $\mathcal{E}$ of approach spaces and defines an approach space to be connected provided it is $\mathcal{E}$-connected in the sense of Preuss. Main results: (1) Products of connected spaces are connected. (2) A topological space is connected if and only if, considered as approach space, it is connected. (3) A pseudometric space is uniformly connected (= Cantor-connected) if and only if, considered as approach space, it is connected. The author then associates with each approach space X a number $\operatorname{con}(X)$ in $[0,\infty]$ as a measure of connectedness of X. Results: (4) X is connected if and only if $\operatorname{con}(X)=0$. (5) $\operatorname{con}\bigl(\prod X_i\bigr)=\operatorname{Sup}\operatorname{con}(X_i)$ for families of nonempty approach spaces. (6) $\operatorname{con}(f[X])\leq \operatorname{con}(X)$ for morphisms $f:X\to Y$ in $\mathbf{AP}$. (7) $\operatorname{con}\bigl(\bigcup X_i\bigr)\leq \sup\operatorname{con}(X_i)$ for families of subspaces of X with nonempty intersection.

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MSC:

54B30 Categorical methods in general topology
54A05 Topological spaces and generalizations (closure spaces, etc.)
54D05 Connected and locally connected spaces (general aspects)

Keywords:

approach space; uniformly connected pseudometric space

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