

**Duffie, Darrell; Singleton, Kenneth J.**

**Simulated moments estimation of Markov models of asset prices.** (English) Zbl 0783.62099  
*Econometrica* 61, No. 4, 929-952 (1993).

This paper provides a simulated moments estimator (SME) of the parameters of asset-pricing models where asset-prices are determined by a state vector  $y_t$  which follows a time-homogeneous Markov process whose transition function depends on an unknown parameter vector  $\beta_0$ . Asset-prices are observed as  $f(y_t, \beta_0)$ , given  $f$ . Simulated observations  $f(y_t^\beta, \beta_0)$  are taken from a process  $\{y_t^\beta\}$  of the underlying economic model.  $\beta$  is the solution of minimizing the distance between sample moments of  $f(y_t, \beta_0)$  and  $f(y_t^\beta, \beta_0)$ .

In section 2, dynamic asset-pricing as an extended version of a known stochastic growth model is described in order to give an illustration of the problems which arise in the use of simulation methods in econometric estimation. Several considerations of the SME estimation of  $\beta$  are discussed. The most important difficulty between the estimation problem with simulated time series and GMM estimation lies in the parameter dependency of the simulated observations  $\{f(y_t^\beta, \beta_0)\}$ .

Section 3 defines the SME. The difficulties in the use of SME and GMM in various articles are pointed out. These difficulties are circumvented by assuming geometric ergodicity as a condition of the state process ensuring that the simulated processes are asymptotically stationary with an ergodic distribution that is independent of the starting values, and by imposing a damping condition on the feedback effect of the parameters on the law of motion of the state process.

Section 4 describes the concept of geometric ergodicity which is then used to prove weak consistency of the SME. Lipschitz and modulus-of-continuity conditions are used to show strong consistency. Various trade-offs in choosing among the regularity conditions leading to weak and strong consistency are discussed in the context of the asset-pricing model. In section 5, the asymptotic distribution of the SME is derived. Asymptotic normality follows from geometric ergodicity of  $\{y_t\}$  and  $\{y_t^{\beta_0}\}$ . Finally, section 6 discusses a variety of extensions of the SME proposed in this paper.

Reviewer: [R.Fahrion \(Heidelberg\)](#)

**MSC:**

- [62P20](#) Applications of statistics to economics
- [60J20](#) Applications of Markov chains and discrete-time Markov processes on general state spaces (social mobility, learning theory, industrial processes, etc.)
- [60J05](#) Discrete-time Markov processes on general state spaces

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**Keywords:**

generalized method of moments; uniform strong law of large numbers; model estimation; asymptotic normality; simulated moments estimator; asset-pricing models; time-homogeneous Markov process; dynamic asset-pricing; stochastic growth model; parameter dependency; geometric ergodicity; weak consistency; modulus-of-continuity conditions; strong consistency; asymptotic distribution

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