Corsini, Piergiulio

Prolegomena of hypergroup theory. (English) Zbl 0785.20032

Supplemento alla Rivista di Matematica Pura ed Applicata. Tricesimo: Aviani Editore. 215 p. (1993).

As recalled by the author in the introduction of his book, hypergroups (multigroups) were introduced by F. Marty in 1934 and independently by H. S. Wall in 1937 as a natural generalization of groups. Since then the investigation into hypergroup theory knew no respite and developed in the large setting of hyperstructures (multialgebras). Corsini's book is an introduction to hypergroup theory. In fact this is an enlarged new edition of the Italian version dating from 1986.

Each section begins with brief historical comments and references and is illustrated with interesting examples. The first section ("Introduction") contains the fundamental definitions and results on hypergroups. The second section ("Homomorphisms") deals with homomorphisms of various types and correlated equivalences. The results of the third section "Complete parts" are presented in the context of the general theory of hypergroups and also in connection of the heart's structure (the heart of a hypergroup H is the intersection of all the subhypergroups of H which are complete parts). Sections four, "Subhypergroups", and five, "Regular hypergroups", deal with some special types of subhypergroups (closed, normal and invertible, conjugable). The hypergroups of type U and those of type C are also briefly presented. In the following four sections, namely "Join spaces, canonical hypergroups and their generalizations", "m-complete hypergroups", "Cogroups", "Cyclic hypergroups", the most important results on some remarkable classes of hypergroups such as m-complete hypergroups, 1-hypergrups, cogroups, cyclic hypergroups, are given. The section "Associativity semihypergroups and hypergroups" presents associativity semihypergroups and hypergroups as well as (partial) τ-groupoids and constructions of partial τ-groupoids. The Cartesian product of semihypergroups is also the object of this section. In the next section constructions of hypergroups and semihypergroups are given. In connection with the hypergroup theory the notion of hyperring, hypermodule, vectorial hyperspace and the principal results concerning them are presented in the section "Hyperrings, hypermodules and vectorial hyperspaces".

The book (which closes with "Index of symbols", "Analytical index" and with the exhaustive bibliography of 384 titles) is self-contained and the organization of the material aims successfully at making proofs as clear as possible. Corsini's book not only gathers a large number of results on hypergroup theory, it also implicitly proposes a great amount of research work.

Reviewer: M.Guţan (Aubière)

MSC:

20N20 Hypergroups
20-02 Research exposition (monographs, survey articles) pertaining to group theory

Keywords:

multigroups; multialgebras; hypergroups; generalization of groups; hyperstructures; introduction to hypergroup theory