Wu, Congxin; Bu, Qingying
Köthe dual of Banach sequence spaces $\ell_p[X](1 \leq p < \infty)$ and Grothendieck space. (English)
Zbl 0785.46009
Commentat. Math. Univ. Carol. 34, No. 2, 265-273 (1993).

Let X be a Banach space and X^* its topological dual. For $1 \leq p < \infty$, let $\ell_p[X] = \{(x_i)\colon \sum_{i\geq 1} |f(x_i)|^p < \infty \ \forall f \in X^*\}$. Then $\ell_p[X]$ is a Banach space for each fixed p, with the norm

$$\|(x_i)\|_{(\ell_p)} = \sup\left\{ \left(\sum_{i\geq 1} |f(x_i)|^p \right)^{1/p} : \ f \in B_{X^*} \right\}$$

where B_{X^*} denotes the closed unit ball of X^*. The Köthe dual is defined as

$$\ell_p[X]^\times = \left\{ (f_i) \in X^* : \sum_{i\geq 1} |f_i(x_i)| < \infty \ \forall (x_i) \text{ in } \ell_p[X] \right\}.$$

The authors establish the following result:

Let $1 \leq p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Then $(f_i) \in \ell_p[X]^\times \Leftrightarrow f_i = \sum_{n\geq 1} r_n s_i^{(n)} h_n$ $(i = 1, 2, \dots)$ for some $(r_n) \in \ell_1$, a bounded sequence $\{s^{(n)}\}$ of sequences of ℓ_q and a bounded sequence (h_n) of X^*.

There is also a necessary and sufficient condition in order that for each fixed p with $1 < p < \infty$, $\ell_p[X]$ is a Grothendieck space.

Reviewer: K.Chandrasekhara Rao (Karaikudi)

MSC:

46A45 Sequence spaces (including Köthe sequence spaces)
46B45 Banach sequence spaces

Keywords:

vector-valued sequence spaces; Köthe dual; Grothendieck space

Full Text: EuDML