Bramson, Maury; **Kalikow, Steven**

Nonuniqueness in g-functions. (English) Zbl 0786.60043
Isr. J. Math. 84, No. 1-2, 153-160 (1993).

Let (X_n) be a one-sided shift with a finite state space A. The conditional expectation $P(X_0 = a_0 \mid X_{-1} = a_{-1},\ X_{-2} = a_{-2}, \dots)$ gives a function $f(a_0 \mid a_{-1}, a_{-2}, \dots)$ with

$$0 \le f(a_0 \mid a_{-1}, a_{-2}, \dots) \le 1, \quad \sum_{a \in A} f(a \mid a_{-1}, a_{-2}, \dots) = 1.$$

Such functions are called g-functions; they were introduced by *W. Doeblin* and *R. Fortet* [Bull. Soc. Math. Fr. 65, 132-148 (1937; Zbl 0018.03303)]. To a g-function there might exist a corresponding stationary process, i.e. a (shift-)invariant measure on A^N. *M. Keane* [Invent. Math. 16, 309-324 (1972; Zbl 0241.28014); for more results see also *H. Berbee*, Probab. Theory Relat. Fields 76, 243-253 (1987; Zbl 0611.60059)] showed that this is the case for continuous g-measures and gave conditions for which the measure is strongly mixing and unique. *B. Petit* [C. R. Acad. Sci., Paris, Sér. A 280, 17-20 (1975; Zbl 0301.28012)] showed that all differentiable g-functions f with

$$\varepsilon \le f \le 1 - \varepsilon \tag{*}$$

for some $0 < \varepsilon < 1/2$ have unique measures which are weakly Bernoulli. *S. Kalikow* [Isr. J. Math. 71, No. 1, 33-54 (1990; Zbl 0711.60041)] noticed that the continuity of a g-function f is equivalent to uniform convergence of the martingales $P(X_0 = a_0 \mid X_{-1}, X_{-2}, \dots, X_{-n})$, $n = 1, 2, \dots$, $a_0 \in A$, where (X_i) is the corresponding stationary process. The martingales converge uniformly if and only if (X_i) is a random Markov chain, i.e. there exist r.v. N_i with values in $\mathbb{N}$ such that (X_i, N_i) is a stationary process, N_0 is independent of $(a_i, N_i)_{i<0}$ and

$$P(X_0 = a_0 \mid X_{-1} = a_{-1}, \dots, X_{-n} = a_{-n}, N_0 = n) = P(X_0 = a_0 \mid (X_i)_{i<0} = (a_i)_{i<0}, N_0 = n) \tag{**}$$

for all n, $(a_i)_{i<0}$.

From the proof of Theorem 7 of the Kalikow's paper one can derive that if a continuous g-function f satisfies (*) and (**) with $EN_0 < \infty$, then the corresponding invariant measure is determined uniquely. The paper under review gives an example of a continuous g- function which satisfies (*) but has two measures.

Reviewer: D.Volný (Praha)

MSC:

60G10 Stationary stochastic processes

Keywords:

g-functions; one-sided shift with a finite state space; conditional expectation; stationary process

Full Text: DOI

References:

[1] M. Aizenman, J. Chayes, L. Chayes and C. Newman,Discontinuity of the magnetization in one-dimensional 1/|x| 2 Ising and Potts models, J. Stat. Physics50 (1988), 1–40. · Zbl 1084.82514 · doi:10.1007/BF01022985

[2] H. Berbee,Chains with infinite connections: uniqueness and Markov representation, Probability Theory and Related Fields76 (1987), 243–253. · Zbl 0611.60059 · doi:10.1007/BF00319986

[3] W. Doeblin and R. Fortet,Sur les chaines a liaisons completes, Bull. Soc. Math. Fr.65 (1937), 132–148. · Zbl 0018.03303

[4] T. Harris,On chains of infinite order, Pacific Journal Math.5 (1955), 707–724. · Zbl 0066.11402

[5] S. Kalikow,Random Markov processes and uniform martingales, Israel Journal of Mathematics71 (1990), 33–54. · Zbl 0711.60041 · doi:10.1007/BF02807249

[6] S. Kalikow, Y. Katznelson and B. Weiss,Finitarily deterministic generators for zero entropy systems, Israel J. Math.79, (1992), 33–45. · Zbl 0768.60074 · doi:10.1007/BF02764801

[7] M. Keane,Strongly mixing g-measures, Invent. Math.16, (1972), 309–324. · Zbl 0241.28014 · doi:10.1007/BF01425715

[8] B. Petit,Schemas de Bernoulli et g-mesures, C. R. Acad. Sci. Paris, Ser. A280 (1975), 17–20. · Zbl 0301.28012