Du, Qiang; **Gunzburger, Max D.**; **Peterson, Janet S.**

Analysis and approximation of the Ginzburg-Landau model of superconductivity. (English)

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In this clearly written paper the authors: (1) sketch the physical derivation of equations governing superconductivity, (2) present an analysis of existence and uniqueness (up to gauge invariance) of their solution, (3) explain how finite element theory can be used to solve the equations, and (4) present estimates of the error incurred in the numerical solution. (Uniqueness is not always assured.) The presentation concentrates largely on so-called "low temperature" superconductors, but they explain how their presentation can be applied to "high temperature" superconductors.

The authors begin with the Ginzburg-Landau free energy density and its consequential functional $\mathcal{E}$ whose minimum determines the state of the sample (superconducting or not). Various properties of superconductors are presented, since the authors presume no familiarity with the subject. They show that $\mathcal{E}$ is invariant under gauge transformations and that it has a minimizer which is unique up to gauge transformation. They derive a set of nonlinear partial differential equations and boundary conditions for a complex scalar "order parameter" ψ and a vector "magnetic potential" $\mathbf{A}$. The vector $\mathbf{A}$ can be taken to have vanishing divergence through a gauge transformation. The equations, when written in non-dimensional form, depend on a positive real parameter κ called the "Ginzburg- Landau" parameter.

The authors re-cast the equations as a pair of Poisson equations depending on κ and apply standard results to show that solutions can be found using finite element methods coupled with continuation methods involving κ. The finite element solutions are proved to converge to the solution of the continuous equations, at least for small κ. Finally, the authors discuss an application to the solution in a single lattice cell with periodic extension to all spaces. Although no computational results are presented, the authors indicate that their approach results in a practical numerical method.

Reviewer: Myron Sussman (Bethel Park)

MSC:

65Z05	Applications to the sciences
65N30	Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
35Q60	PDEs in connection with optics and electromagnetic theory
35J60	Nonlinear elliptic equations

Keywords:

Ginzburg-Landau model; continuation method; Ginzburg-Landau equations; superconductivity; gauge transformations; Poisson equations; finite element methods

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