Brodmann, Markus

A priori bounds of Severi type for cohomological Hilbert-functions. (English) Zbl 0788.14012

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Let F be a coherent sheaf on the projective d-space $\mathbb{P}^d$ and $\delta(F) := \min\{\text{depth} F_x = |x \in \mathbb{P}^d \text{ closed point}\}$. A well known result of Serre (which implies the lemma of Enriques-Severi-Zariski) says that, for $i < \delta(F)$, $H^i F(n) = 0$ for $n \ll 0$. In this paper, the author gives a quantitative version of this vanishing theorem in terms of some invariants of the sheaf such as $\delta(F)$, $h^0 F$, $h^1 F(-1), \dots, h^i F(-i)$, the linear subdimension of F (a notion due to the author) defined as the minimal dimension of a linear subspace containing a point of $\text{Ass}(F)$, etc. Since the bounds given by the author are expressed in a rather complicated way, we shall illustrate his results by stating a particular case: if $0 \leq i < \delta(F)$ and $h^j F(-j) = 0$ for $0 \leq j \leq i$ then $h^i F(n) = 0$ for all $n \leq -i$. The main tool used by the author is the exact sequence of the hyperplane section.

Reviewer: I.Coandă (Bucureşti)

MSC:

14F17 Vanishing theorems in algebraic geometry
14F05 Sheaves, derived categories of sheaves, etc. (MSC2010)
13D40 Hilbert-Samuel and Hilbert-Kunz functions; Poincaré series

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